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Dynamical behaviour of the 'beads' along the magnetic field lines near a rotating black hole

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ABSTRACT
The elements of the cold magnetic driven flows behave like beads on the magnetic field line. The inclination of the field lines at the surface of the disc plays a crucial role on the nature of the magnetically driven outflow. For the non-relativistic case, a centrifugally driven outflow of matter from the disc is possible, if the poloidal component of the magnetic field makes an angle of less than a critical 60° with the disc surface. The collimated flows ejected from active galactic nuclei may probably start from the region near the black hole. We investigate the dynamical behavior of the 'beads' on the magnetic field line start from the disc near a black hole. It is found that the critical angle becomes larger than 60° for the rotating black hole case, which may imply that the flows are easy accelerated in the inner edge of the disk surrounding a rotating black hole.

Key words: Accretion disc, jets, black hole.

1 INTRODUCTION

For ten years, production of magnetohydrodynamic jets by an accretion disc has been considered as a promising paradigm to explain the collimated flows ejected from active galactic nuclei and more recently by young stellar objects(Blandford and Payne 1982; Lovelace 1976 and Camezind 1987). In these models the disc is supposed to be threaded by a vertical magnetic field that extends through a dense corona, the magnetosphere above it becoming filled with plasma which can be flung out centrifugally by the magnetic field lines. In magnetically driven flows, the strength and distribution of the magnetic flux at the disc surface plays an important role as a boundary condition for the problem. The angular momentum loss rate of the flow depends mostly on the strength of the field. The inclination of the field lines at the surface strongly influences the way in which the flow is accelerated. The elements of relatively tenuous gas on the magnetic field lines behave like beads on a rigid wire. Blandford and Payne (1982) point out that the beads can be launched centrifugally from a Keplerian accretion disc if the poloidal field direction is inclined at an angle less than 60° to the radial direction. Cao and Spruit (1994) investigate in some detail how the flow properties change with the field inclination at the disc surface. At low inclination, they find a magnetically driven circulation along the disc surface rather than a high density flow. The jets observed in some active galactic nuclei are believed to be accelerated close to the black hole. In this paper we analyze the dynamical features of the beads along the magnetic field line near a rotating black hole in general relativistic frame. The basic equations for the problem are listed in Sect. 2. Sect. 3 contains a brief discussion.

2 BASIC EQUATIONS

The behaviour of flows flung out centrifugally by the magnetic fields is like beads on rigid wires. If the 'beads' start from rest at the disc surface and are carried with constant angular velocity by the 'wire', then the effective potential for a bead on a wire, corotating with the Keplerian angular velocity at a radius r_d, is

$$\Psi_{eff}(r,z) = -\frac{GM}{r_d}\left[\frac{1}{2}\left(\frac{r}{r_d}\right)^2 + \frac{r_d}{(r^2+z^2)^{1/2}}\right], \tag{1}$$

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where the Newtonian gravitational potential is used in the derivation. After some mathematical derivations, one finds that the equilibrium at $r = r_d$ is unstable if the projection of the wire on the meridional plane makes an angle of less than 60° with the equatorial plane (Blandford and Payne 1982). The critical angle of 60° plays an important role on the jet formation. This mechanical analogy does exhibit the essential dynamical feature of centrifugally driven flows not very close to a compact central object. Here we investigate the dynamical properties of beads on the magnetic field line near a rotating black hole.

The space-time generated by a rotating black hole can be represented by Kerr metric in spherical coordinates

$$ds^2 = -\frac{\triangle}{\rho^2}[dt - a\sin^2\theta d\phi]^2 + \frac{\sin^2\theta}{\rho^2}[(r^2+a^2)d\phi - adt]^2 + \frac{\rho^2}{\triangle}dr^2 + \rho^2 d\theta^2, \tag{2}$$

where

$$\triangle \equiv r^2 - 2Mr + a^2,$$

$$\rho^2 \equiv r^2 + a^2\cos^2\theta,$$

a is the angular momentum per unit mass of the black hole, M is the mass of the black hole (setting $G = c = 1$).

The radial motion of a free particle close to a Kerr black hole is governed by the equation:

$$\frac{d^2r}{d\tau^2} + \frac{1}{2}g^{rr}g'_{rr}\left(\frac{dr}{d\tau}\right)^2 - \frac{1}{2}g^{rr}g'_{\theta\theta}\left(\frac{d\theta}{d\tau}\right)^2 - \frac{1}{2}g^{rr}g'_{\phi\phi}\left(\frac{d\phi}{d\tau}\right)^2 - \frac{1}{2}g^{rr}g'_{tt}\left(\frac{dt}{d\tau}\right)^2 - g^{rr}g'_{t\phi}\frac{d\phi}{d\tau}\frac{dt}{d\tau} = 0, \tag{3}$$

where the metric coefficients g^{rr}, $g_{\mu\nu}$, are given by Eq. (2), and $g'_{\mu\nu} = \frac{\partial g_{\mu\nu}}{\partial r}$.

Since we only intend to investigate the instability condition of the particle rotating with the Keplerian angular velocity near the disc surface, the term $-\frac{1}{2}g^{rr}g'_{\theta\theta}\left(\frac{d\theta}{d\tau}\right)^2$ in Eq. (3) could be omitted, similar to the non-relativistic approach. Combining Eqs. (2) and (3), we obtain the equation describing the radial motion of the beads:

$$\frac{du^{r2}}{dr} + \frac{g'_{rr}}{g_{rr}}u^{r2} + \frac{g'_{\phi\phi}\Omega^2 + 2g'_{t\phi}\Omega + g'_{tt}}{g_{\phi\phi}\Omega^2 + 2g_{t\phi}\Omega + g_{tt}}u^{r2} - \frac{g^{rr}(g'_{\phi\phi}\Omega^2 + 2g'_{t\phi}\Omega + g'_{tt})}{g_{\phi\phi}\Omega^2 + 2g_{t\phi}\Omega + g_{tt}} = 0, \tag{4}$$

where $u^{r2} = \frac{dr}{d\tau}$, $\Omega = \frac{d\phi}{dt}$.

The beads move along a rigid magnetic field line have the constant angular velocity measured by a distant observer $\Omega = \frac{d\phi}{dt}$. For the Keplerian accretion disc, $\Omega = \Omega_K$, given by

$$\Omega_K = \frac{u^\phi}{u^t}, \tag{5}$$

where u^ϕ, u^t are velocities of a free particle moving in a circular Keplerian orbit. Using the four-velocity presented in Bardeen et al. (1972), we finally obtain Keplerian angular velocity measured by a distant observer

$$\Omega_K = \pm\frac{M^{1/2}}{r_d^{3/2} \pm aM^{1/2}}, \tag{6}$$

where the upper sign is for the direct orbit while the lower sign the retrograde orbit, r_d is the radial position of the footpoint of the field line. The radius of minimum stable circular orbit r_{ms} surrounding a rotating black is

$$r_{ms} = M\{3 + Z_2 \mp [(3 - Z_1)(3 + Z_1 + 2Z_2)]^{1/2}\}, \tag{7}$$

where

$$Z_1 \equiv 1 + \left(1 - \frac{a^2}{M^2}\right)^{1/3}\left[(1+\frac{a}{M})^{1/3} + (1-\frac{a}{M})^{1/3}\right],$$

$$Z_2 \equiv \left(\frac{3a^2}{M^2} + Z_1^2\right)^{1/2},$$

the upper sign for the direct orbit.

Integrating Eq. (4), we can get the effective potential for a bead on a magnetic field line corotating with the Keplerian angular velocity Ω_K:

$$\Psi_{eff} = -\frac{1}{g_{rr}} - \frac{C}{g_{rr}(g_{\phi\phi}\Omega_K^2 + 2g_{t\phi}\Omega_K + g_{tt})}, \tag{8}$$

where the integral constant C is determined by the boundary condition

$$\Psi_{eff} = 0, \quad at \quad r = r_d, \quad \theta = \frac{\pi}{2}. \tag{9}$$

Now, the instability condition could then be derived from the second derivative of the effective potential. After some mathematical deductions, we get the critical angle α_{crit} of the magnetic line at the surface of the disc as follows

$$\alpha_{crit} = \arctan\left\{\frac{r_d^2[(2r_d^3 + 4Ma^2)\Omega_K^2 - 8Ma\Omega_K + 4M]}{(2r_d^5 + 2a^2r_d^3 + 8Ma^2r_d^2 + 4Ma^4)\Omega_K^2 - 8Ma(r_d^2+a^2)\Omega_K + 4Ma^2}\right\}^{1/2}. \tag{10}$$

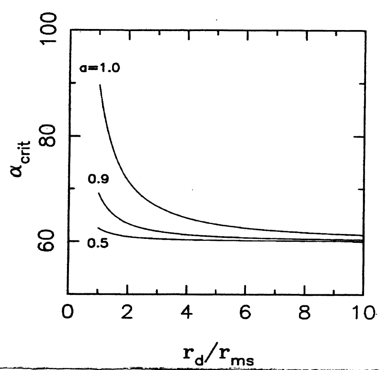

Figure 1. The critical angle θ_{crit} vs. radial position of the magnetic field footpoint at the disc surface with respect to the different rotating black hole angular momentum, $a = 0.5, 0.9, 1.0M$.

In the non-rotating black hole case, $a = 0$, we have $\alpha_{crit} = 60^\circ$, which is same as that obtained by Blandford and Payne (1982) for the non-relativistic case. In Fig. 1 we depict the relations between the critical angle and radial position of the footpoint of the magnetic line for the rotating black hole with different angular momentum a.

3 DISCUSSION

The configuration and strength of the magnetic field of the disc plays a crucial role on the dynamical properties of the centrifugally driven magnetohydrodynamic flow. Assuming that the field configuration above the disc is close to a potential field, the shape of the disc field is determined entirely by the magnetic field B_z at the disc surface. The elements of the cold magnetic driven flows behave like beads on the magnetic field lines. Whether the matter at the disc surface can be flung out mainly depends on the inclination angle of the field line at the disc surface (Blandford and Payne 1982). Blandford and Payne (1982) point out that the flow can be centrifugally driven if the field lines inclined less than the critical angle of 60° with respect to the radial direction. The space near the inner region of the disc is a 'dead zone', where the magnetic field line is too steep, the flow not being able to overcome a barrier of gravitational potential.

We investigate the dynamical properties of the bead on the magnetic fields threading the rotating disc around a Kerr black hole. We have the same critical angle 60° for the non-rotating black hole ($a = 0$) as Newtonian case. Even if in the region very close to a Schwarzschild black hole, the flow could not be centrifugally flung out if the field lines inclined greater than the critical angle 60° with respect to the

radial direction. Abramowicz(1990) point out that the centrifugal force reverses sign at $r = 3M$. For the free particle in circular motion, the minimum radius is $4M$ (unstable). The results obtained here could not be extended to the region $r = 3M$. Cao(1995) study the same problem by using a pseudo-Newtonian potential to simulate general relativistic effects. They find that the critical angle becomes larger in the region close to a non-rotating black hole. The reason is that the angular velocity derived from the pseudo-Newtonian potential simulates $\frac{d\phi}{d\tau}$, not $\frac{d\phi}{dt}$. In general relativistic frame, $\frac{d\phi}{d\tau}$ and $\frac{d\phi}{dt}$ have different values. The beads move along a rigid magnetic field line have a constant angular velocity $\frac{d\phi}{dt}$ measured by a distant observer.

The results are shown in Fig. 1 for the rotating black holes. The critical angle θ_{crit} increases with the reduction of the radius of the magnetic field line footpoint. For the beads at the minimum stable radius of an extreme Kerr black hole, the critical angle θ_{crit} could be as large as 90°, which may imply that the flows could be centrifugally accelerated to high velocities even by the magnetic field lines with low inclination angles very close to the black hole. We note that the critical angle falls rapidly in the cases of relative slowly rotating black hole, for example, $\theta_{crit} \sim 70^\circ$ at the inner edge of the disc for $a = 0.9$. The results obtained here strongly imply that the rotating black hole will be helpful on the centrifugally acceleration of flows very close to the black hole. The 'dead zone' for accelerating over the the inner region of the disc will disappear for the rapidly rotating black hole with an appropriate disc magnetic field configuration. The further investigation on this problem using the full relativistic MHD approach is necessary.

We also study the case that the matter in the disc is in the retrograde orbit around the black hole, though it is not sure whether such disc exists in reality. The results show that the critical angle is slightly influenced by the rotating black hole, the critical angle is always less than, but very close to 60°. The critical angle at the minimum stable radius for the extreme Kerr black hole is about 57°, almost same as the non-rotating case.

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